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**First degree function and**

**second degree function**

School grade: K9

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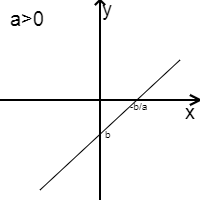
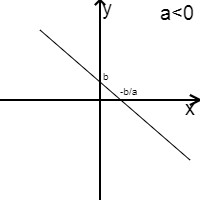
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# **FIRST-GRADE FUNCTION**

**Definition**

The function f:R→R,f(x)=ax+b,a,b∈R,a≠0 is called the first degree function.

The geometric representation of the graph of the first degree function is a straight line.

If a>0 the function is strictly increasing, and if a<0 the function is strictly decreasing.

A function of the first degree is just an ordinary function that has the form:

f(x)=a⋅x+bf(x)=a⋅x+b, where a and b are two real numbers.

Now, it would be good if we had a≠0a≠0, because if it were 00, then we would only have a constant function, of the form f(x)=bf(x)=b, which always returns the same value.

Some examples of first degree functions would be:









## Why "first degree function"?

It all boils down to what power x has. In our case it is to the power of 1, namely



A function of the first degree also has an equation attached:



For example, the following functions each have an equation attached:

 has ecuation 

 will have ecuation , where a is 

Function  is a first degree function with coefficients .

Function   is a linear function with .

Function  is constant function when 

## Properties of the first degree function

A function of the first degree is a linear function in the end. This means that it is represented by a straight line and that it even borrows the properties of such a function. Among which::

The graph of the function of the first degree is a straight line, which has a slope that we can calculate

For an equation , formula for  (slope of the right) is:



And in the case of a function, we will do nothing but replace that one with . Thus, the equation of the line for a function of the first degree will become:



In fact, the slope of the line is the coefficient of x, i.e. a, from the general form of the function 

1. The coordinates of a point on the right of the function also represent a solution for the equation attached to the function.
2. As is normal, the solution of a function like this , represents the coordinates of a point on the graph of the function. This means that those numbers also represent a solution for the equation attached to the function.
3. More precisely, if we have a function  and we will take a value for x, let's say , thn the point  will belong to the graph of the function and will also be a solution for the equation 
4. To represent a function of the first degree, we must find the intersection of the graph with the axes.

Because the graph of this function is a straight line, we need 2 points to represent it correctly. And the easiest points to find out are the intersection of the graph with the axes.

For example, for  , we will have:

* the intersection with the y axis, when ,

meaning  and we will have the point 

* and the intersection with the x axis, when 

namely  it turns out that  and we still have the point

# **Monotonicity of the first degree function**

It is important when we want to learn more about a function, to notice its monotony.

That is, if a function is increasing or decreasing.

The monotonicity of the function of the first degree is given by a, the coefficient of x, namely:

* When a>0 then the function is increasing ↗
* Or when  a<0 the function is decreasing ↘

If we think of f(x) as an equation of a line, then aa would be the slope of the line. More precisely, a is that m in the general form of the equation of a straight line:

f(x)=a⋅x+b or 

y=m⋅x+n or 

And we know that if the slope of the right is a positive number, then the right is increasing (that is, directed to the upper right corner).

***Demonstration***

To test the monotonicity of the function, the rate of increase (decrease) of f will be used,

 for 

If  then f is strictly increasing, and if  then f is strictly decreasing.

### ***Remarks***

1. **The sign of a specifies the monotonicity of the function of the first degree.**
2. The ecuation  represents a slope line  (an oblique line not parallel to the Ox axis or to the Oy axis).

***Exercises:***

State the monotony of the following functions:

1. f(x)=4⋅x

A: the function is increasing, because a>0, namely a=4

2. f(x)=3−5⋅x

A: the function is decreasing, because a<0a<0, more precisely a=−5

3. f(x)=(m−1)⋅x+3

A: in this case, everything depends on m, more precisely when m−1 is smaller or larger than 0.

For example, if we have m−1>0⇒m>1 then f(x) will be increasing, because the number next to x (the coefficient) is greater than 0,

but when m−1<0 or m<1 then f(x) is decreasing.

# **The sign of the function of the first degree**

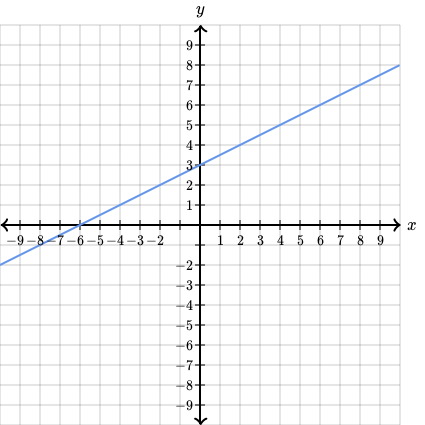
Usually the first degree function is defined on ℝ, that means it extends from −∞ to +∞.

And because the function is represented by a straight line, and most of the time the straight line is oblique, the graph of the function will intersect the Ox axis at a point that will tell us that half of that graph is above the axis, and half below it.

## The values ​​of the function of the first degree

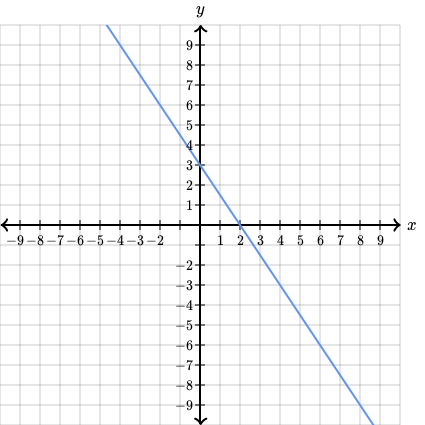
If we want to know what kind of number the function will return, i.e. if it is positive or negative, first of all we can look at the monotony of the function.

If a, the coefficient of x, is positive, then the graph of the function is an increasing line, like this::



And in this case, it is observed that up to the point when x=−6, the function returns negative values ​​(that is, y<0). And after this, it returns only positive values.

If a<0 then the graph of the function will be a decreasing line:



## Exchange point

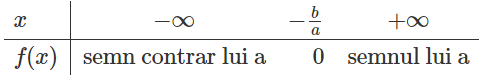
In both cases, we saw that the function will return values ​​with the opposite sign of a up to a point, and after that, with the sign of a.

That point is also called the root of the equation because at that moment y=0.

So to find that point we must have f(x)=0 and if we take the general form of the function::

a⋅x+b=0, we will get the value for x 

So, until  the function has the opposite sign of a and after, the sign of a. We can see this in the following table:



## Practical examples

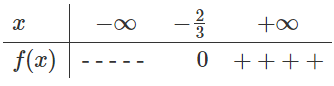
### Find the sign for the following functions:

1. f(x)=3x+2

A: first of all we have to calculate the point where the sign of the function changes, i.e. when f(x)=0

3x+2=0 it turns out that x= , so the sign of the function will be negative until 

and positive after it, as follows:

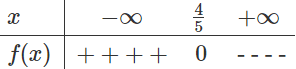


1. f(x)=4−5⋅x

R: we will calculate the point where the sign changes::

−5⋅x=0 ⇒ −5⋅x=−4 ⇒ x= 

and we will have the opposite sign of a up to this point, but because a=−5, the function will be positive on this interval and negative then:



# **Inequalities of the first degree**

For a first degree function like f(x)=7⋅x+8, we can create an inequality of the form 7⋅x+8≥0 (or ≤0).

This inequality is nothing but the expression of the function for which we want to calculate the values ​​of x, which tell us the places where the function is smaller or larger than 0. And when we say that the function is larger than 0, it means that it returns positive numbers.

**Why?**

The main reason to create an inequality from a function expression is to learn more about the function. More precisely, we can find out for which values ​​of x the function will be greater or less than 0.

We don't necessarily have to do this, but only if we are asked or if we are particularly interested in finding out on which interval the function returns positive or negative values.

We can actually imagine that any inequality (of the form a⋅x+b≥0) has an attached function, and when we solve it, we learn something about the function that has the same expression.

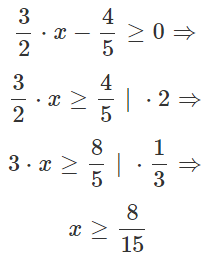
## How do we calculate?

The solution is done in the normal way of calculating an inequality. The interpretation is then more interesting.

For example, let's say we have the function:

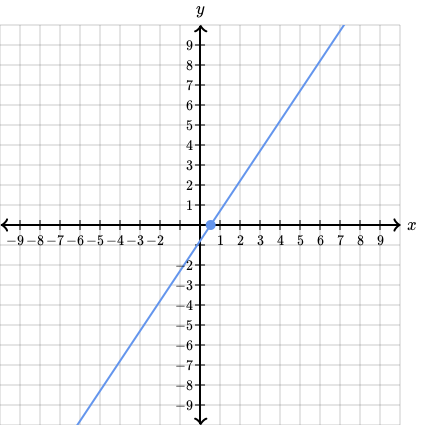


and we will calculate for this function, when its equation is greater than 0, namely::

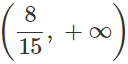


And it turns out that the function f(x) is always greater than 0 when 

So,  is the point after which the function will return only positive values. And if we look at the graph of the function we see that  (almost ) represents the intersection of the graph with the Ox axis, above which, obviously, we will find only positive values.



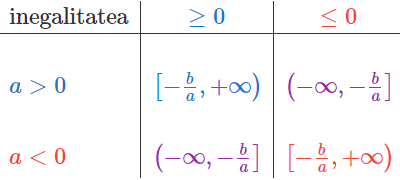
But this point also represents the place where the function changes its sign, something we discussed in the last lesson..

So, the solution for our inequality , is the interval that starts from the intersection of the function with the Ox axis and continues towards +∞, that is .



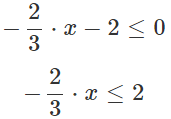
But in general, all solutions for such inequalities start from a point like  and continue to +∞ or −∞. We can deduce a more general definition, as::

The solution of an inequality of the form a⋅x+b≥0a⋅x+b≥0 (or ≤0≤0) is the interval that starts (or ends) with –b/a but depends on a, as follows::

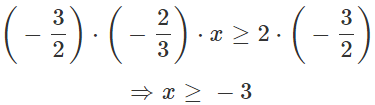


Let's take the following example to see exactly how we use this table..

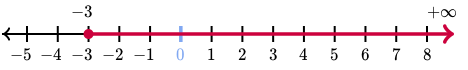
Let's say we have the function  and we want to know when this is ≤ 0.



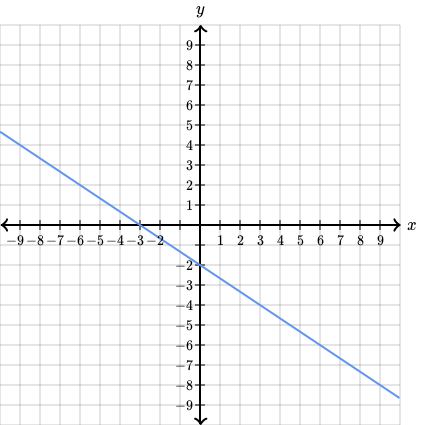
and because a is less than 0, when we multiply by its inverse, the inequality sign will change, namely:



It follows that the solution is the interval **[−3,+∞)**



Therefore, the sign of a matters because it influences the inequality sign. He is also the one who tells us if the graph of the function is increasing or decreasing. In this case it is negative, it means that the function is decreasing, and this is explained by the fact that up to a point we will find positive numbers, and then negative numbers. That is why the solution of the inequality starts from a point (in our case −3) and continues towards +∞.



This can also be seen from the graph of the function, until point −3 we have positive and then negative numbers. Therefore, we can also find the solution of an inequality from the graph of the function.

Sources

https://matematic.eu/LectiaDeMatematica/FunctiaDeGradul1.html

https://www.edumo.org/lectie/6401139935281152 - fctie de gr I

https://www.edumo.org/lectie/6086439091568640 - propr fct gr I

https://www.edumo.org/lectie/5551744788463616 - monotonie

https://www.edumo.org/lectie/5345009758896128 - semnul fctiei de gr I

https://www.edumo.org/lectie/5037387289722880 - inecuatii

https://ro.wikipedia.org/wiki/Func%C8%9Bie\_algebric%C4%83\_de\_gradul\_%C3%AEnt%C3%A2i

II DEGREE FUNCTION

**Definition of the function of the second degree**

*f* : , *f*(*x*)=*ax2*+*bx+c, a*0*, a,b,c*.

**Graphical representation of the second degree function**The graph of the function of degree II is a parabola, having the vertex , where

which is also called the discriminant of the second degree function, and the graph has the right axis of symmetry. 

# **The minimum and maximum of the second degree function. The image of the function of the second degree.**

*-* The function of degree II admits a minimum for (this is also the case for the graph example below) and the minimum value is and is obtained for .

Diagram

Description automatically generated

- The function of degree II admits a maximum for  (it is also the case of the graph example below) and the maximum value is  and is obtained for .

Diagram

Description automatically generated

Regarding the image of the function of the second degree (so the set of its values

*y*=*f*(*x*)=*ax2*+*bx+c*) This is:

if , and respectively if .

# **Monotonia functiei de gradul II**

* Pentru , functia de gradul II admite un minim si este *descrecatoare* pentru si *crescatoare* pentru .



* Pentru , functia de gradul II admite un maxim si este *crescatoare* pentru si

*descrescatoare* pentru .

# **Forma canonica a functiei de gradul II**

Pentru functia de gradul II *se defineste forma sa canonica* ca fiind Text

Description automatically generatedcare ne conduce si la valorile de minim si maxim de mai inainte ca si la obtinerea radacinilor ecuatiei de gradul II, atunci cand , dupa cum vom vedea mai departe.

# **Pozitia parabolei fata de axa Ox. Intersectia graficului cu axele de coordonate. Semnul functiei de gradul II**

* Intersectia cu axa OY este data de punctul de coordonate .
* Intersectia cu axa OX se obtine rezolvand ecuatia *f*(*x*) = 0. Daca , atunci ecuatia *f*(*x*) = 0 are radacini reale:

,

 hence the roots of the equation of degree II *ax2*+*bx+c*=0*, a*0*, a,b,c* are  which are also the abscissas of the points of intersection with the OX axis..

* If then the graph intersects the OX axis at the points A picture containing night sky

  Description automatically generatedand  as can be seen from the following drawings..

Diagram

Description automatically generated Diagram

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As for the sign of the function of degree II, in this case, it is also suggested by the graphs above and obviously it is given by the sign of  and the sign of a. So we have:



*x*

*f*(*x*)

Acelas semn cu *a* 0 Semn contrar lui *a* 0 Acelas semn cu *a*

The graph of the function is located both above and below the OX axis.

* If the graph intersects the OX axis at the point  which is also the peak of the parable.

Diagram

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The sign of the function of degree II, in this case, also suggested by the graphs above, is given by the sign of  and the sign of a is:

The graph of the function is located only above or below the OX axis, having only the top of the parabola on the OX axis..

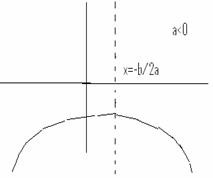
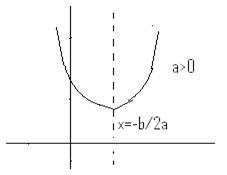


*x*

*f*(*x*)

Acelas semn cu *a* 0 Acelas semn cu *a*

* If  then the graph does not intersect the OX axis and the top of the parabola is above the OX axis (case ) or below it (case ).



The sign of the function of degree II, in this case, also suggested by the graphs above, is given by the sign of  and the sign of *a is*:



*x f*(*x*)

Acelas semn cu *a*

The graph of the function is located just above or below the OX axis.

Note: The sign of the function of the second degree is used to solve the inequality of the second degree, to deduce the sign of a product or a fraction containing functions of the second degree, etc.

# **The relations between roots and coefficients (Viète's relations). The linear form of the second degree function.**

Taking into account the canonical form of the second degree function Text

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we have , from where the relationships between roots and coefficients (Viète's formulas) result:

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Observation.

-The relations between roots and coefficients do not solve the second degree equation. They serve to solve various exercises in which additional relationships related to the roots appear. It is worth noting the way in which various expressions containing the roots are expressed  and  depending on these realities. E.g::

 or

.

-If the two roots are given  and  or the sum S and their product P, then the equation of the second degree from which they came from can be formed:

sau .

# ***Inequalities of form ax2*+*bx+c* 0 (,,), studied on or on intervals of real numbers**

The inequality ***ax2*+*bx+c* 0 (,,)** is solved by constructing the table of the sign for ***f(x)= ax2*+*bx+c***, from where the interval (or intervals) that satisfies (satisfies) the inequality is chosen as the solution of the inequality. If the inequality is solved on intervals of real numbers, then the solution obtained before intersects with the meeting of these intervals, thus obtaining the final solution of the inequality.

# ***Systems of inequalities of the second degree, studied on or on intervals of real numbers***

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Each inequality is solved separately, obtaining the solutions  (for the first inequality),  (for the second inequality),,  (for the *n* inequality). Where is the solution to the system of inequalities obtained (if it is solved on ) as being .



If the system is solved on a meeting of intervals, then the solution it is intersected by the meeting of intervals.



# **Systems of second degree equations**

* 1. **Form systems**

where *a,b,c,d,m,n,p*

in which one equation is of degree I and one of degree II.

From the equation of the first degree, one unknown is substituted according to the other, for example,  and it is inserted into the equation of the second degree, obtaining:

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which solved gives two solutions.

Returning with these values ​​in the substitution relation, the pairs of solutions are obtained

A picture containing dark, night sky

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* 1. **Solving form systems**

, .

also called symmetrical systems.

Taking into account that the relations above can be the relations between the roots and coefficients of an equation of the second degree, the equation is then constructed , which solved gives two solutions 

and from here the solutions of the system are obtained:

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Example. To solve the system in the set of real numbers:



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* 1. **Homogeneous systems**

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Solving these systems is done in the following way: multiply the first equation by  and the second equation with (), so

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By adding the two equations we get the relationship which by dividing by

, it goes tot the equation . Note wth we arrive at an equation of grade II, . Assuming that the solutions of this equation are 

then we can form the systems:

Graphical user interface

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which are obviously systems of type *a*).

# **Exercises**

1. Parameter value  for which the equation  has a distinct solution in the interval  is:
2. a) ; b) ; c) ; d) ; e) .

*Solution: Because the equation* to have a distinct solution in the interval  the conditions must be met simultaneously:

so Text

Description automatically generatedand where



. Which it follows that ,

so answer correct *d*).

1. The real number x is strictly greater than its square, if and only if:

a)  b)  c) d)  e) 

*Solution: . The inequality* has the solution . The correct answer is a).

1. Let the equation , where . If the number is complex  is the root of the equation then:

a)  b) c) d)  e) . 

Solution: Because the coefficients m and n are real numbers and the equation admits the complex root , then the equation admits the root and conjugate . From the relations of Viète si  result and the correct answer is b)

1. For the family of functions of the second degree  the vertices of the associated parabolas are on the right side of the equationecuatie:

**a)** ; **b)** ; **c)** ; **d) **; **e)** .

Solution: The abscissa of the vertex V of the parabola is and the order is

. The result is: which is the equation of the second bisector of the XOY axis system). So **c)**

The set of all values ​​of the real parameter m for which

, is:

**a)** (the crowd empty) ; **b) **; **c)** **d)** ; **e)**.

*Solutions*: The conditions are: si . Results . So **b)**.

1. The set of all parameter values for which the roots of the quation  is for is:



**a) **; **b) **; **c)** (empty crowd) ; **d) **; **e)** .

*Solution*: . From the given condition and from

Result . So **c)**.

1. Let the inequality . Among the following intervals, the set of all solutions of this inequality is:

a) ; b) ; c) ; d) ; e) .

*Solution*: From the conditions of existence . for  the inequality is obviously satisfied. For the by squaring the given inequality we get



.

so the solution . The solution will be

1. Be function , . Actual parameter values for which 

Are:

a) ; b) ; c) ; d) ; e) .

*Solution*: We know if and only if the equation has solution in , if the equation has real



sol. in , so mean



for . The condition that and to be solution of



The sum of the integral solutions of the inequality  is:

a)  b)  c)  d)  e) 

*Solution*: , so , so

Correct answer **c**).

1.  Let be the function  The set of parameter values

for which the graph of the function f intersects the x-axis two distinct points is:

a) b) c)



e)

*Solution*: For the given equation , impose , where does it come from 

Correct answer **a**).

1. The real values ​​of the parameter m, for which are:

a) b) c) d) e) 



the fraction to be positive, it must be , what it implies

pe

, de u

*Solution*: Because



The correct answer is **b**).

1. Function The values ​​of the parameter for which the parabola associated with the function f is tangent to the Ox are:

a) b) c) d) e) 

*Solution*: Equation  must have only one solution, so for  impose . The result is , so

Correct answer **c**).

1. The inequality  has solution:

a) b) c) d)  e) 

*Solution*: The inequality is equivalent to Text

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The solution for this is Correct answer **e**).

1. Image of the function  is:

a) c) d) 



b)

e)

*Solution*: It is verified that as the function f is continuous, so it has



si

Darboux's property, it turns out Correct answer **b**).

The the product value



15) Function

is

a) b) c) d) e)

*Solution*: Equation has roots si , so



, which involves , and the product value Correct answer **d**).