

**Definite Integrals**

School grade: K12

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# Some properties of the indefinite integral

**Theorem 9:** If the functions *f*: *I* →R şi ****: *I* →R admit primitives on the interval I and the function f has a continuous derivative on the interval I, then the following properties hold:

1. 
2. , unde 
3. , where 
4. , where the constant 
5. 
6. , where the constant 
7. , where the constant 
8. 
9. 
10. .

# Table of indefinite integrals (immediate primitives)

1. , where;
2. , where ;
3. , where , şi sau ;
4. , where , ,or ;
5. , where , ;
6. ,;
7. , where or ;
8. , where ;
9. , where ;
10. , where ;
11. , where ;
12. , where ;
13. , where and ;
14. , where ;
15. , where ;
16. , where  or ;
17. , where  or and ;
18. , where  or ;
19. , where  ot and ;
20. , where ;
21. , where ;
22. , where and ;
23. , where and ;
24. , where  and x2
25. , where and x2
26. , where and 
27. , where e and ;
28. , where and ;
29. ; 30) ;

# Integrals of hyperbolic functions

a) ; b) ; c) ; d) .

31) , where;

32) , where;

33) , where ;

34) , where or ;

35) , where \{1}.

36) +*C*, where .

38) +*C*, where .

# The theorem and formulas for integration by parts

**Theorem 10:** If the functions ***f*:*I* →*R*** şi ***g*:*I* →*R*** are differentiable and have a continuous derivative on the interval *I*, then the functions şi admit primitives on the interval I and the formula holds:

 (1)

**Remark:**If we note şi , then formula (1) takes the more useful form:

 (2)

**• The formulas (1) and (2) they are called the formulas of integration by parts for the indefinite integral.**

# Theorems and integration formulas by change of variable (substitution) for the indefinite integral

**Theorem 11 (the first change of variable formula):** If the function is differentiable on the interval I, and the function  accepts range primitives *J and* is a function primitive , then the function  is a function primitive  and the formula occurs



**Substitution method for calculating the indefinite integral (first variable change method)::**

a) If the substitution is made 

= .

b) If the substitution is made 

= .

**Theorem 12: (second change of variable formula):** If the function is bijective and differentiable on the interval *I,* the function  accepts range primitives *J and* *H* is a function primitive , then the function  supports primitives and function  is a primitive of the function f and the formula occurs.



**Remark:** Practically, when calculating the indefinite integral by the second variable change method, it is noted , where it is from and the formula is applied:

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# The physical meaning of the indefinite integral

**I physical meaning of the indefinite integral:**

• If a mobile (material point) moves non-uniformly and the law of its displacement is , and the law of its velocity variation is , then:

1) From the physical meaning of the derivative we have: ;

2) From the physical meaning of the indefinite integral we have: 

**II physical meaning of the indefinite integral:**

• If a mobile (material point) moves unevenly and the law of its movement is , the law of variation of its speed is , and the law of variation of its acceleration is , then:

1) From the physical meaning of the derivative we have: sau ;

2) From the physical meaning of the indefinite integral we have: ;

3) From the physical meaning of the indefinite integral we have: .

**III physical meaning of the indefinite integral:**

If a material point moves along the Ox axis under the action of the force , then the law of the variation of the work done under the action of the force *F* is : .

# Test

Calculate the indefinite integrals:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Nr | | Variant I | Variant II | Variant III |
|  |  | |  |  |
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